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RESONANT PERIOD OF FREE CORE NUTATION – ITS OBSERVED CHANGES AND EXCITATIONS

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ABSTRACT
The motion of Earth's spin axis in space is monitored by Very Long-Baseline Interferometry (VLBI), and since 1994 also its rate is measured by Global Positioning System (GPS). From the direct analysis of the combined VLBI/GPS solution in the interval 1994.3-2004.6 we recently found that the apparent period of the Retrograde Free Core Nutation (RFCN) grew from original 435 days to 460 days during the past ten years, but the resonance effects yielded a stable period of about 430 days. Now we repeat the same study with VLBI-only data, covering much longer interval (1982.4 – 2005.6). Direct analysis shows again a substantial increase of the apparent period during the last decade or so. The resonant period is given by internal structure of the Earth (mainly by the flattening of the core), so it is highly improbable that it is so much variable. From the same observations we derive corrections of certain nutation terms. A subsequent study of indirect determination of resonance RFCN period from the observed forced nutation terms through the resonance effects proves that the natural resonance period remains stable and is equal to 430.32±0.07 solar days. From this follows that an excitation by outer layers of the Earth (atmospheric, oceanic) should exist, with a terrestrial frequency close to that of RFCN (of about –1.0050 cycles per solar day, i.e. with period of $-23^{h}53^{m}$ mean solar time), invoking the apparent changes of the directly observed RFCN period. Thanks to a close proximity of the resonance, any excitation with this period is extremely amplified so that the excitation necessary to explain the difference can be very small. The atmosphere alone contains enough power to excite the observed changes.

KEYWORDS: geodynamics, Earth orientation, Free Core Nutation, space techniques, VLBI

1. INTRODUCTION

Retrograde Free Core Nutation (RFCN) is a small periodic motion of the Earth's axis of rotation due to the existence of its fluid core. The fluid core can rotate around an axis that is slightly inclined with respect to the axis of rotation of the mantle (see Fig. 1). The axis of Earth's angular momentum keeps the same orientation in space, provided there is no external torque acting on the rotating Earth. All these three axes lie in one plane that slowly rotates clockwise in space around the axis of Earth's angular momentum. Any displacement of the axis of the core thus induces a displacement (much smaller) of the axis of the mantle and vice versa. The motion of the axis of rotation of the mantle is observed by modern space techniques, such as Very Long-Baseline Interferometry (VLBI). This motion is described by offsets of the instantaneous celestial pole from its position defined by conventional IAU precession-nutation model, so called celestial pole offsets.

The resonance period of RFCN in space is dominantly given by the shape of the core; the period is inversely proportional to its dynamical flattening. If the core were in hydrostatic equilibrium, the period would be approximately 460 days – this is the value

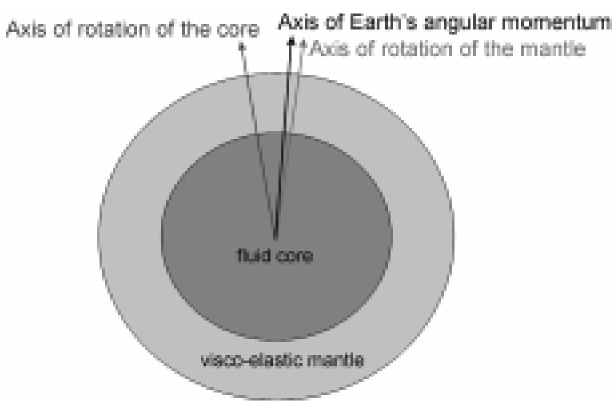

Fig. 1 In absence of external torques, the axis of Earth's angular momentum is fixed with respect to inertial space. The axis of rotation of the core is slightly inclined to axis of rotation of the mantle whose motion is observed by modern space techniques.

used by Wahr (1981) in his model of nutation IAU1980. Later observations by VLBI nevertheless revealed that the core's flattening is slightly larger (by about 4 per cent), leading to RFCN period of about 430 solar days. The value 430.21 was chosen to calculate the recently adopted model of nutation IAU2000 (Mathews et al., 2002, further referred to as MHB). Now more observations with better accuracy are available which enables a new study of this problem. In order to distinguish between the period obtained from the Fourier analysis of the time series of observed celestial pole offsets and the resonance period which is given by the internal structure of the Earth, we shall use the term "apparent period" in the first case and "resonance period" or simply "period" in the second one.

Recently we used the combined solution of VLBI and GPS observations in the interval 1994.3 – 2004.6 to study RFCN (Vondrák et al., 2005). To this end, we used the method of combined smoothing (Vondrák and Čepek, 2000) and combined the celestial pole offsets measured by VLBI with their rates measured by GPS (Vondrák and Ron, 2005). We found from the direct analysis of celestial pole offsets an increase of the apparent period from 435 to 460 days during the interval studied. However, from the indirect analysis (using the resonance effects on the forced terms of nutation) we found that the resonance period remained very stable throughout the whole interval, and was equal to 430.55 ± 0.11 solar days. Similar results were obtained from VLBI analysis also by other authors (Roosbeek et al., 1999 or Hinderer et al., 2000), who concluded that the RFCN period is stable within 3 days, and is close to 430 solar days. It is worth mentioning that Haas and Schuh (1996) found from the analysis of VLBI-observed tidal displacements of the stations 426±20 sidereal days, while Florsch and Hinderer (2000) found the period of about 428 days from the analysis of tidal gravity data. Dehant et al. (2003) proposed that the difference between the direct and indirect determination of RFCN period may be due to additional excitation produced by external parts of the Earth (oceans, atmosphere). In the following, we make the analysis of VLBI-only observations, covering much longer interval, and propose a possible excitation mechanism responsible for this apparent controversy. It is a continuation of our above cited preceding study (Vondrák et al., 2005) which contains the details concerning the theoretical background and the full description of data analysis that is used also here.

2. DATA USED AND THEIR DIRECT ANALYSIS

We use the combined solution provided by the International VLBI Service for geodesy and astrometry (IVS), denoted as ivs05q3X.eops (Schlueter et al. 2002). This solution uses individual VLBI datasets from different networks and analysis centers. It is assumed to be more robust than the VLBI results of individual VLBI networks and analysis centers as it is less vulnerable to failure than any individual member of the assembly of VLBI systems/VLBI networks/VLBI analysis centers used to define the combined solution (Cannon, 2001). The solution, comprising non-equidistant data with steps ranging from 1 to 7 days, covers the interval 1979.6 – 2005.6. The celestial pole offsets $\delta X, \delta Y$ are given with respect to the model of precession-nutation IAU2000A. They are displayed in Fig. 2, where also the smoothed curves are shown to depict their obvious periodic character. Formal errors of these data continually decrease in time, from the original 0.1-0.3 mas at the beginning to 0.02-0.04 mas at the end of the interval studied. The data are further 'cleaned' to get rid of outliers – all offsets greater than 1 mas are removed as well as the data before 1982.4 (i.e., the data outside the boxes in the figure). The figure also demonstrates the ever-increasing precision of the observations – practically all rejections are made for data observed before 1990.

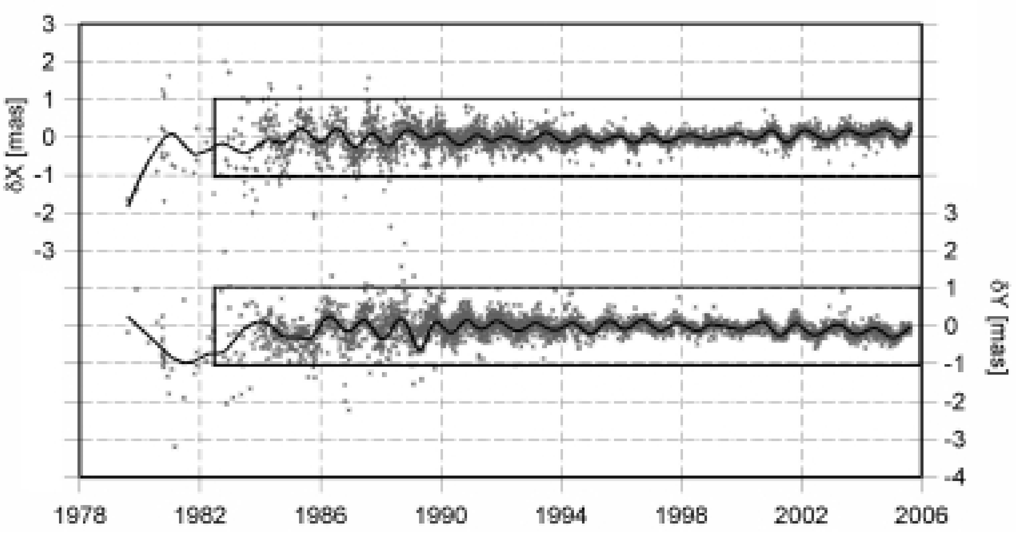

Fig. 2 Celestial pole offsets measured by VLBI (IVS combined solution). Only the data inside the boxes are used.

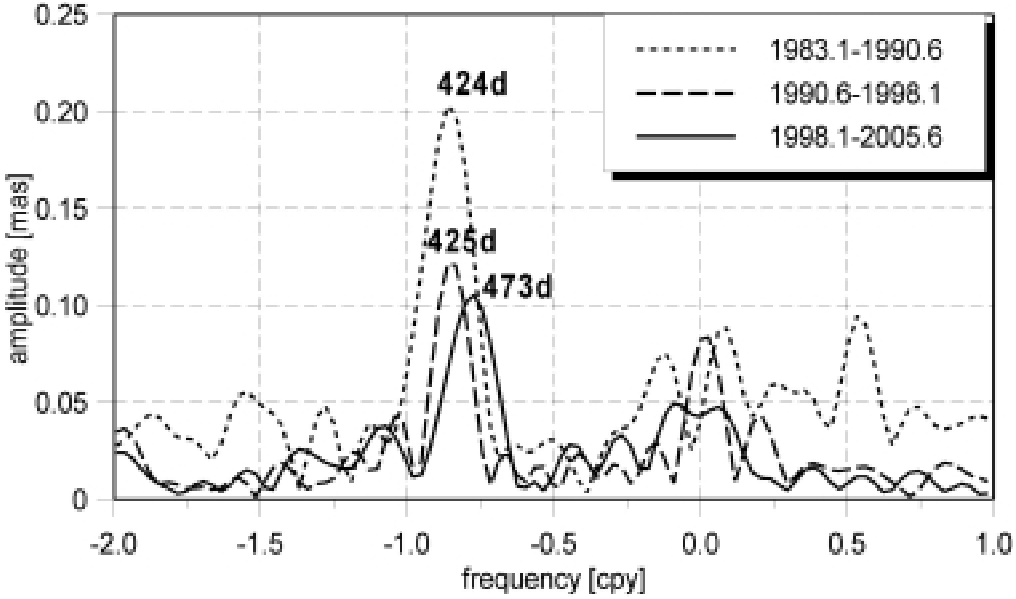

Fig. 3 Spectral analysis of IVS celestial pole offsets in three time windows.

In order to show how the spectrum changes in time we first interpolated the unevenly spaced data to 3-day intervals, using a very weak smoothing (with weights inversely proportional to the squared sigmas of IVS data), followed by linear interpolation. Then the data were divided into three time windows, each 7.5y long. The FFT amplitude spectra of these three series are shown in Fig. 3. All three of them display a similar dominant peak near the RFCN frequency; however, its amplitude decreased in time, while the apparent period substantially increased, especially during the past ten years or so. There is also a very small retrograde annual term which is however hardly visible in the figure because of its proximity to the dominant RFCN peak. Because of the presence of this term, there is certain possibility that the directly determined value of apparent RFCN period from FFT analysis might be biased, as demonstrated by Brzezinski and Petrov (1999). In order to avoid this, we use another method in which both terms are determined simultaneously (see next section).

3. INDIRECT DETERMINATION OF RFCN PERIOD

Theoretically, in absence of any additional excitation, the resonance period of RFCN should remain constant, and its amplitude should exponentially decrease. Because of the resonance, the amplitudes of forced nutation terms of a realistic non-rigid Earth model differ from those corresponding to the solution for the rigid Earth. If the resonance frequency changes the amplitudes of the forced nutation terms should also change. This is especially true for the terms that are close to the resonance. The closest term is the one with retrograde annual period (see below); therefore we look at this very term and its possible variations.

We use the least squares method, applied to 3-day interpolated data (see preceding section), in 6-year sliding intervals, to estimate the parameters of RFCN and retrograde annual term – amplitude and apparent period of the former and only the amplitude of the latter term. The 6-year interval was chosen since it is approximately equal to the beat period between RFCN and retrograde annual term, which should assure that both terms are resolved correctly. To get the full value of the amplitude of annual term (in complex form), we must add the estimated parameters to the values as given in the IAU2000 model. Each nutation term is generally an elliptic motion in celestial frame, and it can be expressed as a sum of two circular terms with different amplitudes and the same frequencies with opposite signs. If the estimated sine/cosine terms in X, Y are s_X, c_X and s_Y, c_Y respectively, the corresponding complex corrections of the prograde and retrograde part of the forced nutation term are

$$\begin{aligned}\Delta C^{+} &= \tfrac{1}{2}\left[c_X + s_Y + \mathrm{i}\,(c_Y - s_X)\right]\\ \Delta C^{-} &= \tfrac{1}{2}\left[c_X - s_Y + \mathrm{i}\,(c_Y + s_X)\right].\end{aligned} \qquad (1)$$

The obtained results are displayed in Figs. 4 and 5.

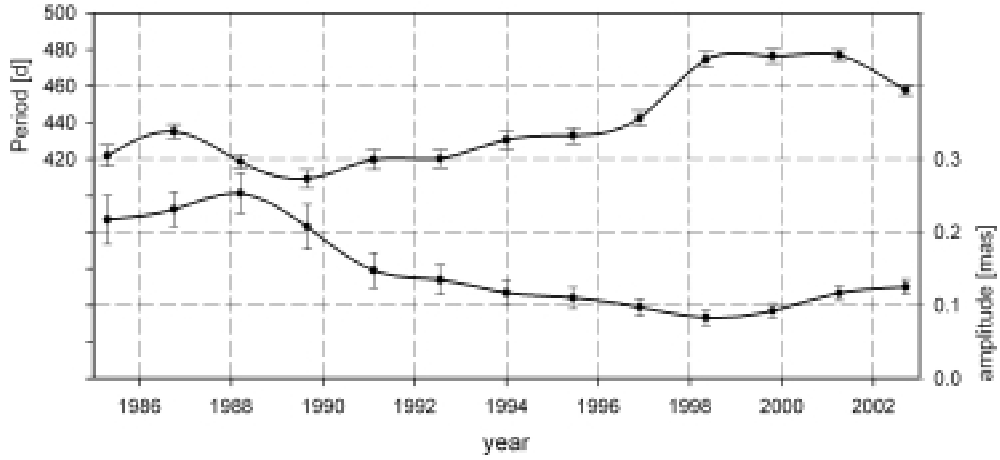

Fig. 4 Observed variation of the apparent period (upper graph) and amplitude (lower graph) of RFCN.

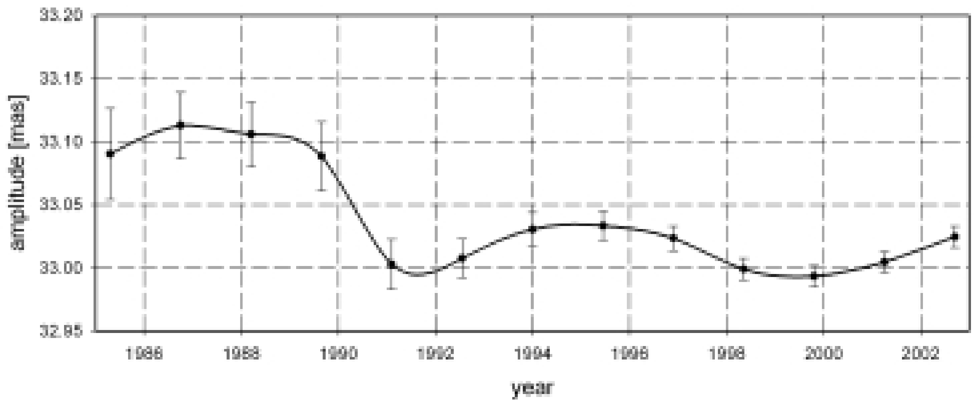

Fig. 5 Observed variation of the amplitude of retrograde annual nutation term.

The changes of RFCN parameters, depicted in Fig. 4, confirm the results obtained by the spectral analysis made in preceding section. It is clear that the retrograde annual term amplitude (see Fig. 5) also changes, but the question arises if the magnitude of these changes is so large that it can be caused by the observed variations of the apparent RFCN period. The dependence of the amplitude of any forced nutation term on the frequency of RFCN can be computed from the MHB transfer function, used to derive the IAU2000 model of nutation,

$$T(\sigma) = \frac{e_R - \sigma}{e_R + 1} N_0 \left[1 + (1+\sigma)\left(Q_0 + \sum_{j=1}^{4} \frac{Q_j}{\sigma - s_j} \right) \right]. \quad (2)$$

The complex function $T(\sigma)$ expresses the ratio of the non-rigid amplitude of a forced nutation term with terrestrial frequency σ (in cycles per sidereal day – cpsd) to its rigid Earth value. Here e_R denotes the dynamical ellipticity of the rigid Earth used to compute the rigid solution, N_0, Q_j are complex constants, and s_j are four complex resonance frequencies corresponding to Chandler Wobble (CW), RFCN, Prograde Free Core Nutation (PFCN) and Inner Core Wobble (ICW) respectively. In our case, only s_2 is interesting since it is the RFCN frequency, closest to the motion we are studying. The real part of the MHB transfer function is depicted in Fig. 6, together with some forced nutation terms; they always appear in pairs, prograde and retrograde, in celestial frame. They are all retrograde in terrestrial frame, placed symmetrically with respect to –1 cpsd.

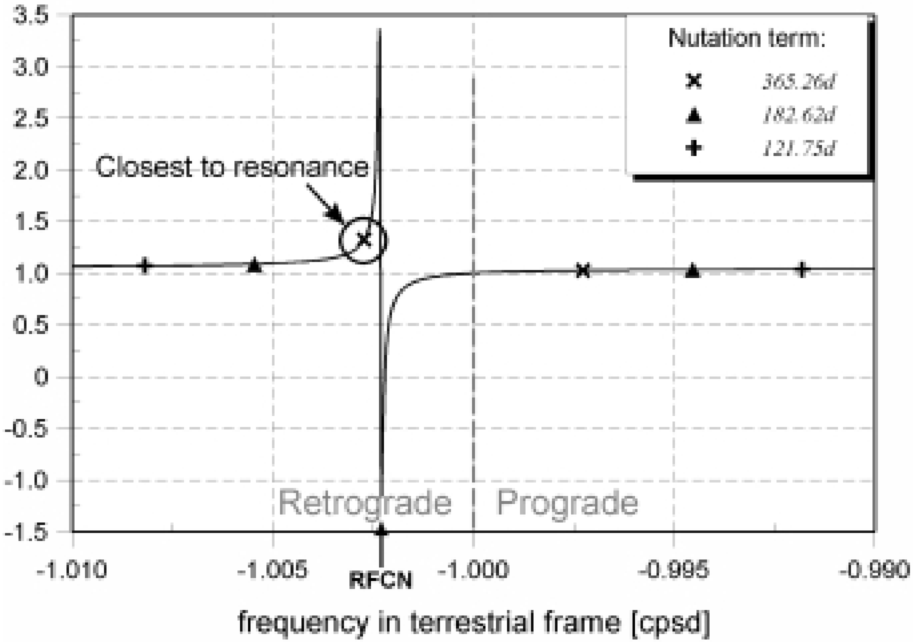

Fig. 6 Real part of MHB transfer function $T(\sigma)$ and some closest forced nutation terms.

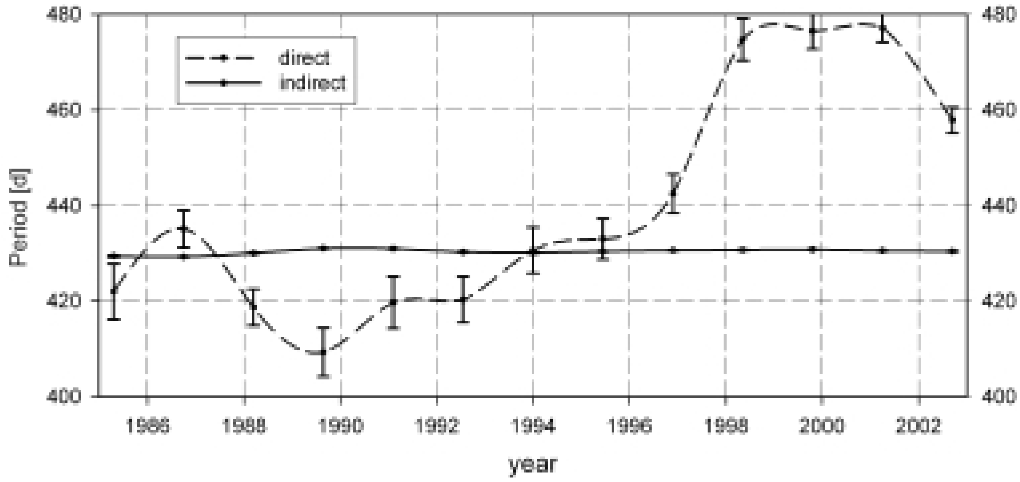

Fig. 7 Period of RFCN – comparison of its apparent and resonance value determined by direct and indirect method respectively.

Now if the resonant frequency of RFCN changes, the curve representing $T(\sigma)$ shifts horizontally, and the value $T(\sigma)$ for any forced term with frequency σ also changes. It is clear that the term that is most sensitive to this change is the retrograde annual term since its frequency is closest to resonance. Knowing the amplitude of this term from the observations, and having its rigid value we can compute the transfer function and then use Eq. (2) inversely, to calculate the resonant frequency s_2. We did so, for all observed values of the retrograde annual term shown in Fig. 5, and displayed the result in Fig. 7. For comparison, we reproduce here also the results of the direct determination shown already in Fig. 4. Direct and indirect determination is depicted as dashed and solid line respectively; standard errors are given by vertical segments. The figure clearly demonstrates that the indirect determination of the resonance period is much more precise, and at the same time much more stable than the direct determination of the apparent period. This is caused by the fact that in the latter case the time variable excitations by out layers of the Earth are present that change both amplitude and phase of the motion in this near-resonance frequency region.

Generally speaking, any complex terrestrial frequency s [cpsd] can be easily converted to celestial period P [solar days] and quality factor Q for this frequency (characterizing the damping of the motion) by using simple relations

$$P = 0.99727/[\mathrm{Re}(s)+1], \quad Q = -\mathrm{Re}(s)/2\,\mathrm{Im}(s) \qquad (3)$$

If we accept that the resonant frequency is constant in time, we can use the observed amplitudes of several forced nutation terms (namely those with periods 365.26, 182.62, 121.75, 27.55 and 13.66 days) to estimate a single value of the resonant frequency. We first determined the individual values of transfer function T for these periods (both prograde and retrograde), by dividing the observed amplitudes by their theoretical values for the rigid Earth. Before doing so, we removed from the observed amplitudes small corrections, listed in Table 7 of MHB, that are not due to the response of non-rigid Earth to luni-solar and planetary torques. Then we estimated two unknown parameters $\mathrm{Re}(s_2)$, $\mathrm{Im}(s_2)$ by the method of least squares to fit the expression (2) to these 'observed' values T. A more detailed description of the procedure used can be found in our previous paper (Vondrák et al. 2005). Thus we arrive at

$$s_2 = -1.00231749 + 0.00002433\mathrm{i} \pm 40(1+\mathrm{i})10^{-8} \text{ [cpsd]}, \qquad (4)$$

leading to the value of the resonance RFCN period equal to $P_f = -430.32 \pm 0.07$ solar days and quality factor $Q_f = 20600 \pm 340$, by using Eq. (3). The latter value corresponds to a damping time (i.e., the time necessary to reach half-amplitude of the free motion) of about 12 years.

4. LOOKING FOR EXCITATION

The main difference between the direct and indirect determination of RFCN apparent and resonance period respectively is that

1. The direct determination from observed celestial pole offsets provides a sum of free motion (with resonance period) with the forced motion due to changes in the outer layers of the Earth (oceans, atmosphere);
2. The indirect determination through resonances provides only the resonant period which is given by the internal structure of the Earth (mainly the flattening of the core).

The inevitable conclusion is that the observed changes of the apparent RFCN period coming from the direct determination must be caused by an additional excitation. This excitation should have a frequency close to that of RFCN, i.e. about –1.0023 cpsd, or –1.0050 cycles per solar day in terrestrial frame. But how large should be its amplitude? Here Brzezinski's (1994) so called broad-band Liouville equation in frequency domain will help. It expresses the dependence of the amplitude of polar motion p on the amplitude of excitation χ for a given frequency σ, in complex form. Here we use only the atmospheric pressure term χ_P since the influence of wind term is two orders of magnitude smaller and so completely negligible:

$$p(\sigma) = \chi_P \left[\frac{\sigma_{CW}}{\sigma_{CW} - \sigma} + \frac{9.509 \times 10^{-2} \sigma_{CW}}{\sigma_{FCN} - \sigma} \right]. \qquad (5)$$

$\sigma_{CW}, \sigma_{FCN}$ are the complex terrestrial frequencies of Chandler wobble and RFCN, respectively. Graphical representation of the expression in brackets for retrograde celestial periods of the excitation ranging from 430 to 460 days is given in Fig. 8.

We used the resonance RFCN frequency σ_{FCN} equal to s_2, as given by our analysis above (Eq. (4)), Chandler frequency σ_{CW} was put equal to the MHB value used in their resonance formula (s_1 of their Table 6), i.e., $0.00311279 + 0.00037610\mathrm{i}$ [cpsd]. In

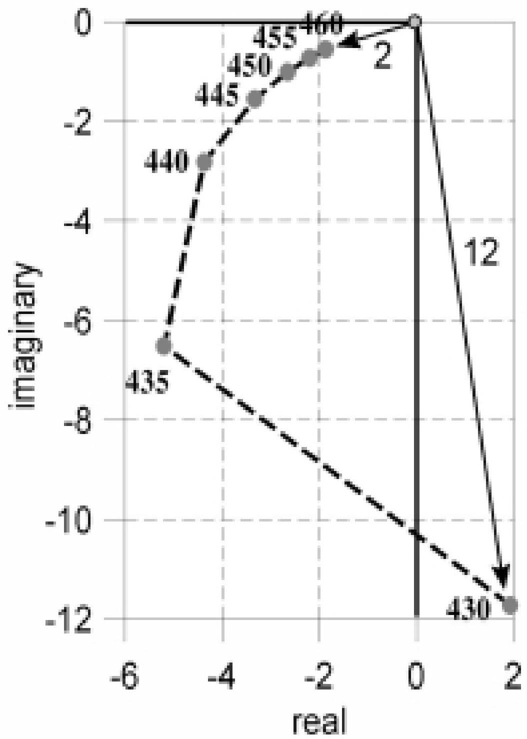

Fig. 8 Complex ratio of the amplitude of polar motion to atmospheric pressure excitation, calculated for different celestial periods of the excitation.

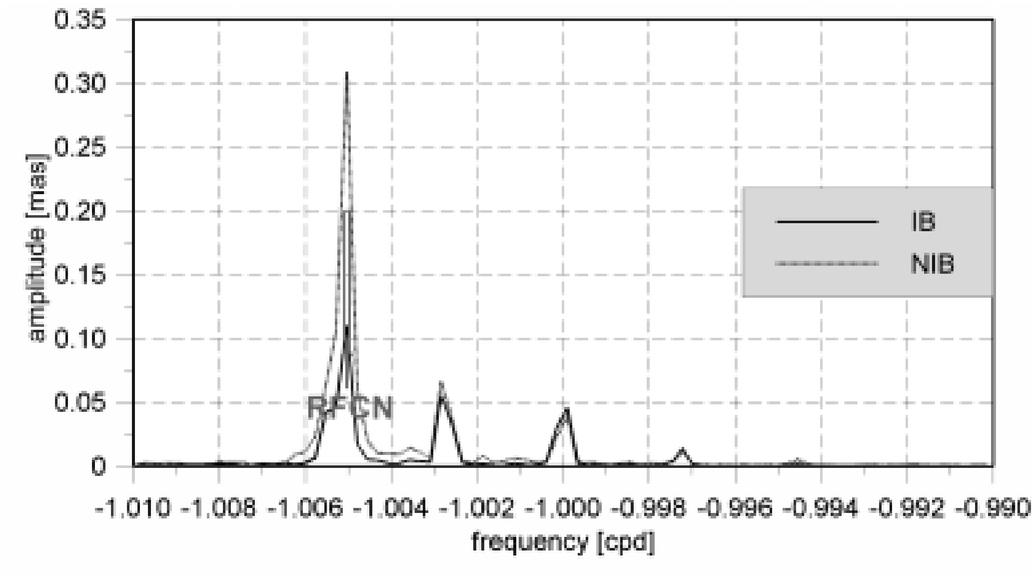

Fig. 9 Spectrum of AAM pressure term with (IB) and without (NIB) inverted barometer correction, convoluted with Brzezinski transfer function.

fact, any similar value of σ_{CW} can serve well for our purpose since it is very far from RFCN resonance and thus unimportant. Gray circles connected by straight dashed lines display the complex values of the expression (i.e., the transfer function between the amplitude of the pressure term and polar motion) for different values of the period. Absolute value of the transfer function ranges from 2 to 12, and the phase also changes substantially. Thus to produce 0.1 mas amplitude of polar motion at nearly diurnal retrograde frequency (or the forced nutation close to RFCN) the excitation of only 10–50 μas is needed.

There are atmospheric angular momentum functions (AAM) available from the IERS Special Bureau for the Atmosphere with sub-diurnal resolution (in 6-hour intervals) from the NCEP/NCAR re-analysis of atmospheric data (Salstein, 2005); oceanic data are publicly available only in daily intervals. Therefore we use only AAM to estimate if there is enough power to produce the observed changes near RFCN frequency. To this end, we calculated the FFT spectral analysis of AAM pressure data and convoluted the amplitude spectrum with Brzezinski's transfer function as given in Eq. (5). The result is depicted in Fig. 9, where only a part of the spectrum around RFCN in terrestrial frame is shown. Two convoluted spectra are given there – with inverted barometer correction (IB) and without it (NIB).

The figure clearly shows that the required power is available, especially in NIB series; the peak is even slightly higher than the observed amplitude of motion near RFCN, represented by a rectangular box in Fig. 9 (its width corresponds to the observed range of period, its height to the observed range of amplitude), and also AAM with IB correction probably contains enough power.

5. CONCLUSIONS

The resonance period, given by the internal structure of the Earth, is relatively stable in time, as determined by the indirect analysis through observed forced nutation terms. It is stable within a fraction of a day, as demonstrated by Fig. 7 and Eq. (4). The value that we obtained here from 23 years of VLBI data (–430.32±0.07 solar days) is in a good agreement with MHB, with our previous determination (Vondrák et al., 2005), and also with the findings of other researchers (Roosbeek et al., 1999; Florsch and Hinderer, 2000; Hinderer et al., 2000 or Dehant et al., 2003). The observed large changes of the apparent period, obtained from the direct analysis of celestial pole offsets, can be most probably ascribed to an additional excitation by external parts of the Earth (atmosphere, oceans). The terrestrial period of the necessary excitation must be close to –23h 53 min; its amplitude of about 10–50 μas (i.e., close to the noise level of AAM data) is sufficient to produce the observed changes. Even the atmospheric pressure term alone (without inverted barometer correction) meets this condition.

ACKNOWLEDGMENTS

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